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THE PROBLEM OF CURRENT LEADS IN SUPERCONDUCTING DEVICES

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A method is described for determining the heat flux along electrical leads to a cryostat with liquid helium. Formulas are obtained which enable the heat flux to the liquid helium to be calculated.

Recently, superconductors have been described that are capable of retaining their superconducting properties in very strong magnetic fields. The smaller the liquifier (or condenser) required to cool the working medium (helium) in normal operation, the greater the advantage of using superconductors as windings in electrical engineering devices, both as regards efficiency and size.

The liquefier (condenser) power is determined by the total heat flux to the cryostat, which must be drawn off to maintain the windings at the boiling point of the coolant (for helium -4.2°K). Heat is withdrawn from the cryostat at the expense of the latent heat of vaporization of the liquid helium supplied continuously from the liquifier. The heat which must be withdrawn from the cryostat is composed of the heat flux through the cryostat insulation, and through the supports on which the superconducting winding is mounted; the heat liberated in the winding itself (e. g. , dielectric losses in the insulation of the leads; in the structural elements due to Foucault (eddy) currents when the winding has an ac supply, and also during transient dc processes; and of the heat flux traveling along the electrical leads due to the thermal conductivity of the lead material; ohmic losses in the nonsuperconducting part of the leads.

Calculation of the heat flux through the leads and supports is particularly difficult because of the need to take into account the variation of the material properties along their length, and also the cooling of the leads and supports by gas (helium) from the cryostat. We shall attempt to solve the problem with the aid of certain assumptions.

One end of the electrical lead along which a current I flows is at temperature τ_2, and the other at a higher temperature τ_1. The lateral surface of the lead is cooled by outflowing gaseous helium. The length of lead between points with the respective temperatures τ_1 and τ_2 is l. The wire radius is r and its cross-sectional area S. The heat flow is along the lead in the direction of the x coordinate. The resistance of the lead material as a function of temperature is described by the relation $\rho = \rho(\tau)$. The form of this function is known. The mean value of the resistance on the section (τ_1, τ_2) is

$$\rho_m = \frac{1}{\tau_2 - \tau_1} \int_{\tau_1}^{\tau_2} \rho(\tau)\, d\tau.$$

If we relate this resistance to the center of the section and assume the variation of the resistance to be linear, then at temperature τ_x the resistance will be

$$\rho_x = \frac{2\tau_x}{(\tau_2 - \tau_1)^2} \int_{\tau_1}^{\tau_2} \rho(\tau)\, d\tau = K_\rho \tau_x. \quad (1)$$

The thermal conductivity at a point with temperature τ_x is related to ρ_x by the Wiedemann-Franz law:

$$\rho_x \lambda_x = 3\left(\frac{k}{e}\right)^2 \tau_x. \quad (2)$$

Substituting the value of ρ_x from (1) into (2), we obtain as a first approximation for λ_x

$$\lambda_x = \frac{3}{2}\,\frac{k^2}{e^2}\,\frac{(\tau_2 - \tau_1)^2}{\int_{\tau_1}^{\tau_2} \rho(\tau)\, d\tau} = \text{const} = \frac{1}{\tau_2 - \tau_1} \int_{\tau_1}^{\tau_2} \lambda(\tau)\, d\tau. \quad (3)$$

Formulation of the heat balance equation

The heat flux through the cross section of the lead at the points x and x + dx is

$$q_x = -\lambda \frac{d\tau}{dx} S, \quad (4)$$

$$q_{x+dx} = -\lambda \frac{d}{dx}\left(\tau + \frac{d\tau}{dx}dx\right) S. \quad (5)$$

The difference

$$-dq = q_x - q_{x+dx} = \lambda \frac{d_2\tau}{dx^2} dx\, S \quad (6)$$

is determined by the heat created in the element dx and the heat withdrawn from it by gaseous helium:

$$dq = I^2 \rho_x dx/S + nc_p d\tau. \quad (7)$$

This representation of the heat withdrawn by the gaseous helium is equivalent to assuming perfect heat transfer between the electrical leads and the gas, which, as shown in [5], does not lead to any appreciable error in the case of boiling liquid helium.

Equating the right sides of (6) and (7) and using (1), we obtain a second-order differential equation for determining the temperature along the lead

$$\frac{d^2\tau}{dx^2} + \frac{nc_p}{\lambda S}\frac{d\tau}{dx} + \frac{I^2 K_\rho}{\lambda S^2}\tau = 0. \quad (8)$$

Replacing the coefficients of $d\tau/dx$ and τ by A and B, we obtain

$$\frac{d^2\tau}{dx^2} + A\frac{d\tau}{dx} + B\tau = 0. \quad (9)$$

Solution of the equation and temperature distribution along the lead

Let us examine a number of specific cases.

(a) When $\xi^2 = A^2 - 4B^2 > 0$

$$\tau = C_1 \exp\left[\frac{-A+\xi}{2}x\right] + C_2 \exp\left[\frac{-A-\xi}{2}x\right]. \quad (10)$$

and (b) when $\xi^2 = 4B^2 - A^2 > 0$

$$\tau = D \exp\left[-\frac{1}{2}Ax\right] \sin\frac{1}{2}\xi(x-E). \quad (11)$$

The condition $4B^2 - A^2 > 0$ is physically real, but it is not possible to obtain a steady oscillating temperature field. The parameter E (as can quite easily be proved) can only be varied so that part of a sinusoid fits into the length of the lead.

(c) When $4B^2 = A^2$

$$\tau = \exp\left[-\frac{1}{2}Ax\right](C_1 x + C_2). \quad (12)$$

The boundary conditions are: at $x = 0$ $\tau = \tau_1$, at $x = l$ $\tau = \tau_2$

The constants of integration are:
for case (a)

$$C_1 = \frac{\tau_2 - \tau_1 \exp[(-A-\xi)l/2]}{\exp[-Al/2]\, 2\,\mathrm{sh}(\xi l/2)}, \quad C_2 = \tau_1 - C_1; \quad (13)$$

for case (b)

$$D = \tau_1 \left\{1 + \left(\frac{\tau_2 + \tau_1 \exp[-Al/2]\cos(\xi l/2)}{\tau_1 \exp[-Al/2]\sin(\xi l/2)}\right)^2\right\}^{-\frac{1}{2}},$$

$$E = -\frac{2}{\xi}\,\mathrm{arctg}\left\{\frac{\tau_2 + \tau_1\exp[-Al/2]\cos(\xi l/2)}{\tau_1 \exp[-Al/2]\sin(\xi l/2)}\right\}; \quad (14)$$

and for case (c)

$$C_1 = \frac{\tau_2 - \tau_1 \exp[-Al/2]}{\exp[-Al/2]}, \quad C_2 = \tau_1. \quad (15)$$

The heat in flux due to a current lead located in gaseous helium is

$$q_c = -k_q \left(\frac{d\tau}{dx}\right) \lambda S =$$

$$= k_q \left\{\frac{A-\xi}{2} C_1 \exp\left[\frac{-A+\xi}{2} l\right] + \frac{A+\xi}{2} C_2 \exp\left[\frac{-A-\xi}{2} l\right]\right\} \lambda S.$$

Here $k_q \cong 1.5$ is an empirical factor allowing for the nonuniform temperature distribution over the lead section and for nonideal heat transfer from lead to gaseous helium. This factor is required in the subsequent refinement on the basis of experimental data.

The heat flux to liquid helium due to losses in nonsuperconducting lead wires immersed in liquid helium is

$$q_b = \rho_2 l_2 I^2 / S.$$

The total consumption of helium in the cryostat by evaporation due to the principal heat fluxes is

$$V = (q_a + q_b + q_c)/\rho'.$$

The fall in helium level in the cryostat by evaporation due to heat flux from the leads is

$$\Delta h = 4\rho' (q_b + q_c) / \pi D^2.$$

The total heat flux to the liquid helium is

$$\Sigma Q = q_a + q_b + q_c.$$

Knowing the heat flux ΣQ, we can refine the quantity $n = 0.0107\ \Sigma Q$, and effect a corresponding refinement of the values of A, ξ, C_1 and C_2. The heat flux to the liquid helium determines the rate of evaporation, i.e., the required output of the helium liquefier.

Numerical example of evaporation of helium-1 due to heat flux from current leads

Fifty (N = 50) copper conductors enter a cylindrical cryostat (D = 5 cm) containing helium-1 (boiling point $\tau_2 = 4.2°K$), each carrying a current I = 0.5 a. The diameter of each conductor 2r = 0.1 mm, and the length l = 25 cm, not counting 18 cm in the liquid helium. The other ends of the leads are at the boiling point of liquid nitrogen ($\tau_1 = 77.4°K$). In the absence of leads the helium level in the cryostat falls by Δh = 12 cm in T = 7 hr.

The volume of helium evaporating per hour in the absence of leads is

$$V = \frac{\Delta h}{T} \frac{\pi D^2}{4} = \frac{12}{7} \frac{\pi}{4} \cdot 25 = 0.0337\ l.$$

This corresponds to a heat flux through the insulation $q_a = V\rho' = 0.0337\ l/\text{hr} \cdot 1.37\ l\text{W} \cdot \text{hr} = 24.6 \cdot 10^{-3}$ joule/sec. To the heat flux q_a it should be added that due to the copper conductors immersed in liquid helium, the latter are not superconducting. At 4.2° K their electrical resistivity is $\rho_2 = 7.4 \cdot 10^{-9}$ ohm · cm.

The power released in the copper conductors immersed in liquid helium is

$$q_b = N \rho_2 l I^2 / S = 21 \cdot 10^{-3}\ \text{W}.$$

The number of moles of gas evaporating per second due to the heat flux $q_a + q_b$ is

$$n = 0.0456 \cdot 0.0107 = 4.88 \cdot 10^{-4}\ \text{mole/sec}.$$

For helium-1 c_p is 21.2 J/mole · °K.

The mean value of the thermal conductivity in the temperature range 4.2°K - 77.4°K is

$$\lambda = \frac{1}{77.4 - 4.2} \int_{4.2}^{77.4} \lambda(\tau)\, d\tau = 9.64\ \text{W/cm} \cdot °\text{K}.$$

The factor K_ρ is given by

$$K_\rho = \frac{2}{(77.4 - 4.2)^2} \int_{4.2}^{77.4} \rho(\tau)\, d\tau = 1.77 \cdot 10^{-9}\ \text{ohm/cm} \cdot °\text{K}.$$

Constants A and B by

$$A = 0.278, \quad B = 7.4 \cdot 10^{-3}.$$

Solution of the differential equation (9) gives

$$\xi = \sqrt{A^2 - 4B^2} = 0.27259,$$

$$C_1 = \frac{\tau_2 - \tau_1 \exp[(-A-\xi)\, l/2]}{\exp[-Al/2]\, 2\mathrm{sh}(\xi\, l/2)} = -228.6,$$

$$C_2 = \tau_1 - C_1 = 306.0,$$

$$\frac{d\tau}{dx} = \frac{-A+\xi}{2} C_1 \exp\left[\frac{-A+\xi}{2} l\right] + $$
$$+ C_2 \frac{-A-\xi}{2} \exp\left[\frac{-A-\xi}{2} l\right] \cong -1.46.$$

The heat flux along the leads is

$$q_c = -k_q \lambda S \frac{d\tau}{dx} = 84 \cdot 10^{-3} \ \mathrm{W}.$$

The total power released in the cryostat is

$$\Sigma Q = q_a + q_b + q_c = (24.6 + 21 + 84) \cdot 10^{-3} \cong 0.13 \ \mathrm{W}.$$

Since the amount of cooling gas in the calculation was taken on the basis not of ΣQ, but only of $q_a + q_b$, it is necessary to refine the value of q_c by carrying out the calculation with the new value of n. This gives q_c = 46 mW. The final value of q_c is 60 mW. The experimental value of q_c for the case computed is 57 mW [5]. The discrepancy between the experimental value and values obtained by calculation is mainly due to the fact that in the calculations incomplete heat transfer between gas and conductor was allowed for only approximately with the aid of the factor k_q = = 1.5. When the surface of the conductor is developed, the error due to this assumption will be reduced.

The drop in helium level due to evaporation is

$$\Delta h = 4V/\pi D^2 = 4\Sigma Q \rho'/\pi D^2 = 7.5 \ \mathrm{cm/hr}.$$

NOTATION

τ_1 and τ_2 – temperatures of "hot" and "cold" parts of lead; λ – thermal conductivity; $\rho(\tau)$ – exact dependence of electrical resistivity on temperature in the temperature range examined; k – Boltzmann constant; e – electronic charge; n = 0.0107 ΣQ – number of moles of gas evaporated per second [4]; c_p – molar specific heat of gas; ΣQ – heat flux to cryostat; ρ_2 – electrical resistivity of lead material located in liquid helium; ρ' – specific heat of vaporization of liquid helium-1; D – inside diameter of cylindrical cryostat; q_a – sum of heat fluxes due to first four causes mentioned at beginning of paper.

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